PROPAGATION OF SPHERICAL AND CYLINDRICAL BLAST WAVES IN A NONHOMOGENEOUS ATMOSPHERE WITH COUNTERPRESSURE TAKEN INTO ACCOUNT

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UDC 534.222.2

We present an approximate method for calculating the propagation of a weak spherical or cylindrical shock wave (with counterpressure taken into account) into a nonhomogeneous exponential atmosphere. In the case of a cylindrical wave with an arbitrary orientation of the cylinder axis the three-dimensional problem is reduced to a two-dimensional one upon introducing the principle of planar sections, i.e., motions of the gas along the cylinder axis are neglected. By means of a parametrization with respect to the positional angle the two-dimensional problem is reduced to a one-dimensional one. To solve the one-dimensional problem, we use the method of "parallel layers": the atmosphere is partitioned into a number of parallel layers of small thickness in each of which the atmosphere may be considered to be homogeneous, and the passage of the wave through a boundary of the layers may be regarded as a passage across the boundary separating two media.

The problem of the propagation of strong cylindrical and spherical blast waves into a nonhomogeneous atmosphere has been considered repeatedly by many authors [1-5]. However, not infrequently, situations arise in practice wherein counterpressure can no longer be neglected, i.e., the wave cannot be considered to be strong.

A classical example of a case of this kind was the flight and explosion of the Tunguska meteorite [6, 7], which resulted in the appearance of a quasicylindrical ballistic wave and a spherical blast wave whose joint effect was the continuous uprooting of a forest over an area of 2000 km^2. The majority of authors estimate the height of the point of explosion in the range of 5 to 10 km, the energy of the explosion in the range of 10 to 40 megatons or $(4 \text{ to } 10)\cdot 10^{23}$ ergs. Under these conditions the blast wave reaches the surface of the earth considerably weakened so that the excess pressure $\Delta p/p_1 < 1$.

The problem so stated is not self-similar. We propose to solve it here by an approximate method, one which we shall refer to as the "method of parallel layers."

We consider first the propagation of a spherical wave. We denote the height of the point of explosion by H_0, the height of the homogeneous atmosphere by H^*, the angle which the direction of propagation of the wave front makes with the vertical by θ, and the distance from the point of explosion by r_2 (Fig. 1). Although the problem is, strictly speaking, two-dimensional, it may be reduced to a one-dimensional problem by means of a parametrization with respect to the positional angle θ. As long as the wave may be regarded as strong, i.e., as long as $p_2/p_1 \geq 40$ [8], we apply one of the solutions for strong waves [1-4]. For the further treatment of the problem, we proceed in the following way.

We partition the atmosphere into a series of parallel layers of thickness ΔH. We consider the atmosphere inside each layer to be homogeneous, and for the propagation of the wave into it, we apply one of the approximate formulas derived for the case of propagation of a weak shock wave into a homogeneous

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 84-90, May-June, 1972. Original article submitted November 10, 1971.

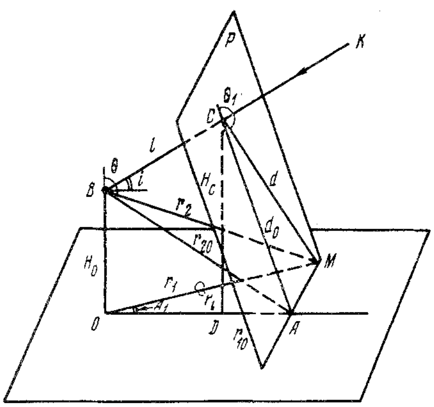

Fig. 1

atmosphere.* Formulas of this kind, for example, were derived in [8]; they give good agreement with the exact results obtained by numerically integrating the differential equations [9].

We now change over to dimensionless variables, expressing the distance from the point (axis) of the explosion as a fraction of a dynamical length r_0

$$r_0 = (E / p_{10})^{1/\nu}, \quad \xi = r / r_0 \tag{1}$$

where E is the energy of the explosion (energy per unit length in the case of the cylindrical wave), p_{10} is the undisturbed air pressure at the height of the explosion, and $\nu = 2$ or 3 for a cylindrical or a spherical wave, respectively. Also, we let q be the relative excess pressure at the jump

$$q = (p_2 - p_1) / p_1 \tag{2}$$

We assume that in the homogeneous atmosphere there exists between q and ξ a relationship of the form $q = q(\xi)$, which we consider as known. We consider the passage of the wave through the boundary of two layers as a passage through the boundary of two media. In addition, both the pressure p_2 at the wave front as well as the pressure p_1 of the undisturbed gas increase by means of a jump. Denoting by p_{21} and p_{11} the values of p_2 and p_1 after the shock wave has passed through the separation boundary and letting $h = \Delta H / H^*$, then for p_{11}, we obtain

$$p_{11} = p_1 e^h \tag{3}$$

The pressure at the wave front, p_{21}, and the quantity $q_1 = (p_{21} - p_{11})/p_{11}$ corresponding to it, may be obtained by solving the problem concerning the decomposition of an arbitrary discontinuity arising as the result of an interaction of a shock wave with the boundary of separation of the layers, which may be regarded as a contact discontinuity. This problem was solved by L. V. Ovsyannikov [10]. Based on his solution,

$$q_1 = q + \psi(q)(e^h - 1) \tag{4}$$

where we have introduced the notation

$$\psi(q) = p_1 \frac{\partial}{\partial p_{11}} \left(\frac{p_{21}}{p_{11}} \right)_{p_{11} = p_1} \tag{5}$$

For $\psi(q)$, L. V. Ovsyannikov obtained the expression

$$\psi(q) = -\frac{2(1+\mu q)\sqrt{1+q}\,[\sqrt{1+(1-\mu)q}\,\sqrt{1+q} + (1-\mu)q]}{2(1+\mu q)\sqrt{1+(1-\mu)q} + (2+\mu q)\sqrt{1+q}} \tag{6}$$

where $\mu = (\gamma + 1)/2\gamma$. For $q < 1$, with an error not exceeding 1%,

$$\psi(q) = -\tfrac{1}{2}[1 + (2 - \mu)q] \tag{7}$$

We introduce an effective relative distance ξ_*, which we define as that value of ξ, at which the excess pressure q_*, in the case of propagation of a standard shock wave into a homogeneous atmosphere with pressure p_{10} and density ρ_{10} of the undisturbed gas, would be equal to the excess pressure q in a real shock wave propagating into a nonhomogeneous exponential atmosphere ($q_* = q(\xi_*)$), where the form of the function $q(\xi_*)$ is the same as the form of the function $q(\xi)$ in the standard wave.

We derive an expression relating ξ_* and ξ. To do this, we form the difference $(\xi_1 - \xi)_n$, accumulating as a result of the nonhomogeneity of the atmosphere with the passage of the wave through the n-th atmosphere layer of thickness h. We note, moreover, that $h \ll 1$, and that, we can then put $e^h = 1 + h$

$$(\xi_1 - \xi)_n = (q - q_1)\frac{d\xi_*}{dq} = -\psi(q)\, h \frac{d\xi_*}{dq} \tag{8}$$

*No exact solutions have as yet been found for this case since, with counterpressure taken into account, the problem becomes nonself-similar.

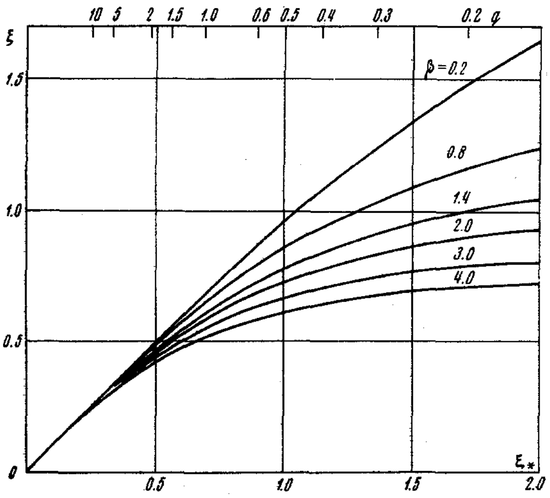

Fig. 2

We divide both sides of Eq. (8) by h and denote the limit of the resulting expression as $h \to 0$ by $f(\xi_*)$:

$$f(\xi_*) = -\psi(q)\frac{d\xi_*}{dq} \tag{9}$$

The difference $\xi_* - \xi$ is obtained by summing Eq. (8) over the layers, with account being taken of Eq. (9). Passing to the limit for $h \to 0$, and recalling that

$$d\xi = \frac{dr}{r_0} = \frac{dH}{r_0 |\cos\theta|} \tag{10}$$

we replace the summation by an integration

$$\xi_* - \xi = \int_{H_0}^{H_n} \frac{f(\xi_*)}{H^*} dH = \frac{r_0 |\cos\theta|}{H^*} \int_{\xi_0}^{\xi_n} f(\xi_*) d\xi \tag{11}$$

In Eq. (11) the integration is with respect to ξ, not with respect to ξ_*. To change to the variable ξ_*, we differentiate Eq. (11), separate the variables, and integrate once again. We obtain

$$\xi = \int_{\xi_0}^{\xi_n} \frac{d\xi_*}{1 + \beta f(\xi_*)} \tag{12}$$

where, we have introduced the notation

$$\beta = (r_0 |\cos\theta|) / H^* \tag{13}$$

Equation (12) can also be readily obtained from the differential equation derived in [10]

$$\frac{d\xi_*}{d\xi} = 1 + \psi(q)\left[\frac{dq}{d\xi_*}\right]^{-1} \frac{d}{d\xi} \ln p_1(\xi) \tag{14}$$

since for the exponential atmosphere

$$\frac{d}{d\xi} \ln p_1(\xi) = -\beta$$

and the first factor on the right side of Eq. (14) is equal to $-f(\xi_*)$.

In the case of an arbitrarily oriented cylindrical wave, when the cylinder axis is inclined at an angle i to the horizontal plane, the problem becomes, in fact, a three-dimensional one. To reduce it to a two-dimensional problem, we use, as was done earlier [5] in obtaining a solution for a strong wave, the law of planar sections, i.e., we consider the propagation of the wave, generated at the point B on the cylinder axis, in the plane P passing through B and perpendicular to the cylinder axis (Fig. 1). In this plane, the quantity $\Delta = H^* \sec i$ plays the role of an effective unit of height; it is to be substituted in place of H^* in all the formulas. Consequently, we have

$$\beta = \frac{r_0}{H^*} \cos i \, |\cos\theta_1| \tag{15}$$

where θ_1 is the positional angle, reckoned in the plane P from its intersection with the vertical plane passing through the axis of the explosion. By parametrizing with respect to the angle θ_1, we reduce the problem to a one-dimensional one. The meaning and the value of the parameter β are clarified below.

An approximate formula, expressing the relationship $q(\xi)$ and, in the given case, $q(\xi_*)$, was derived in [8]; it has the form

$$q = \frac{m}{x_k - 1} \qquad (k = 1, 2, 3) \tag{16}$$

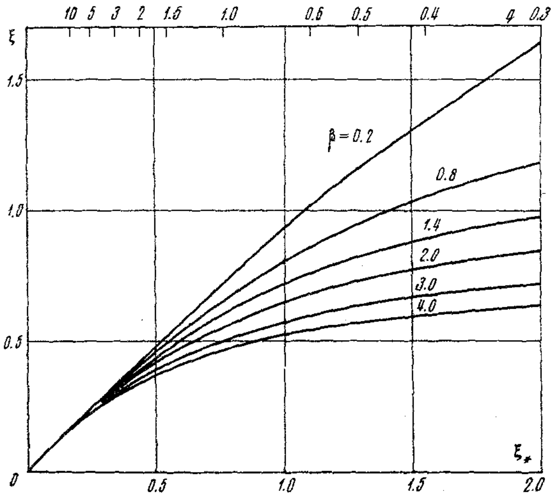

Fig. 3

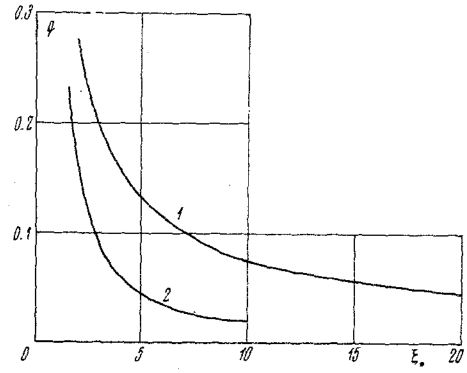

Fig. 4

where we have adopted the notation

$$x_1 = \sqrt{1 + n\xi_*^{\nu}} \qquad (\xi_* \leqslant 2)$$

$$x_2 = \sqrt{1 + n\sqrt{2}\xi_*^{3/2}} \qquad (\xi_* > 2,\ \nu = 2)$$

$$x_3 = \sqrt{1 + 2n\xi_*^2\,[\ln \xi_*/2 + 1]} \qquad (\xi_* > 2,\ \nu = 3) \tag{17}$$

$$m = 4\gamma\ /\ (\gamma + 1),\ n = (\nu + 2)^2\gamma\alpha_\nu,\ \alpha_2 = 0.983,\ \alpha_3 = 0.85$$

The inverse relation $\xi_*(q)$ for k = 1 and 2, may be expressed in the explicit form

$$\xi_* = n^{-1/\nu}\left[\frac{m}{q}\left(\frac{m}{q} + 2\right)\right]^{1/\nu} \qquad (k = 1)$$

$$\xi_* = (2n^2)^{-1/3}\left[\frac{m}{q}\left(\frac{m}{q} + 2\right)\right]^{2/3} \qquad (k = 2) \tag{18}$$

For k = 3 ($\xi_* > 2$, $\nu = 3$) this relation is transcendental; ξ_* can then be obtained by the method of successive approximations.

On the basis of Eqs. (9), (16), and (17), we obtain the following expressions for the functions $f(\xi_*)$ in all three cases:

$$f_1(\xi_*) = \frac{2\psi(q)}{m\nu n}\xi_*^{1-\nu}x_1(x_1 - 1)^2$$

$$f_2(\xi_*) = \frac{4\psi(q)}{3\sqrt{2}\,mn}\xi_*^{-1/2}x_2(x_2 - 1)^2 \tag{19}$$

$$f_3(\xi_*) = \frac{\psi(q)}{2mn}\xi_*^{-1}\left[\ln\frac{\xi_*}{2} + \frac{3}{2}\right]^{-1}x_3(x_3 - 1)^2$$

The form of the functions $f_k(\xi_*)$ shows that the integral (12) cannot be expressed in terms of elementary functions but must be evaluated numerically. With the aid of this integral, we can determine, for an arbitrary combination of γ, ν, and β, the ξ corresponding to a given ξ_*, and then, after expressing the dependence $\xi(\xi_*)$ graphically, we can use it to perform the inverse process. Knowing ξ_*, we can apply at the point in question the approximate formulas derived for a shock wave propagating in a homogeneous atmosphere with counterpressure taken into account [8, 11], or we can use the corresponding tables given in [12, 13], and so obtain all the characteristics of the shock wave, and the gas behind the shock wave.

The parameter β is a measure of the influence of atmospheric nonhomogeneity on the propagation of the shock wave; we speak of it, therefore, as the nonhomogeneity parameter. Indeed, when $\beta = 0$, it follows from Eqs. (11) and (12) that $\xi_* = \xi$, i.e., we have the case of a homogeneous atmosphere. The relationship of ξ with ξ_* for various values of β, for spherical and cylindrical waves, respectively, is shown in Fig. 2 and Fig. 3 (for $\xi_* \leq 2$). Since ξ_* is a single-valued function of q, we have displayed the q values on the upper scale in Figs. 2 and 3. The dependence of q on ξ_* for spherical (curve 1) and cylindrical (curve 2) waves for large ξ_* and small q is shown in Fig. 4.

The dimensionless shock wave propagation velocity v for $\xi_* \leq 2$ may be determined from the expression

$$v \equiv \frac{D}{c} = \frac{\xi_*^{-\nu/2}}{\sqrt{n}}(1 + x_1) \tag{20}$$

where D is the dimensional velocity, c is the sound speed, and $x_1 = \sqrt{1 + n\xi_*^{\nu}}$. Equation (20) is valid for $\xi_* \leq 2$; for $\xi_* > 2$, we have in place of it the following:

for a spherical wave:

$$v = (2n)^{-1/2}\xi_*^{-1}[\ln \xi_* / 2 + 1]^{-1/2}(1 + x_3) \tag{21}$$

for a cylindrical wave:

$$v = (n\sqrt{2})^{-1/2}\xi_*^{-3/4}(1 + x_2) \tag{22}$$

The dimensionless time of passage of the wave is given by

$$\tau = \tau_0 + \gamma^{-1/2}\int_{\xi_0}^{\xi_n} \frac{d\xi}{v} \tag{23}$$

where τ_0 and ξ_0 are related by the equation

$$\tau_0 = \xi_0^{(\nu+2)/2}\alpha^{1/2} \tag{24}$$

The transition from τ to the dimensional time t is effected through use of the formula

$$t = \tau r_0 \left(\frac{\rho_1}{p_1}\right)^{1/2} \tag{25}$$

The method presented here makes it possible to estimate quickly the excess pressure at the front of a weak spherical or cylindrical shock wave propagating from the top downwards in a nonhomogeneous exponential isothermal atmosphere of small height; an estimate of the propagation speed can also be made. A nonisothermal atmosphere can be readily accommodated by expressing the altitude scale for H* as a function of the altitude itself.

The author thanks L. V. Ovsyannikov, V. P. Korobeinikov, L. A. Chudov, and Kh. S. Kestenboim for a valuable discussion of the problem.

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